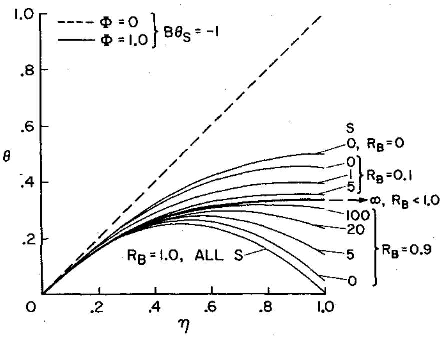

Fig. 2 Steady-state temperature distribution of a weakly-absorbing diffuse reflector—specified heat-transfer coefficient, stream temperature and interface temperature.

decreases with increasing S while for high R_B the temperature increases with increasing S. In both cases as S increases the temperature gradient at $\eta = 1$ decreases. For all distributions shown the conductive heat fluxes are identical at $\eta = 0$ because of the specification $B\theta_s = -1$. Thus, it is evident that as S increases, the steady-state distribution for each family is characterized by decreased conductive energy loss; therefore the radiation absorbed is reduced. This may be described as a consequence of increasing volume reflectance with increasing scattering coefficient.

A case of particular interest is that of zero conduction to the back surface. We may solve (12) for the value of $B\theta_s$, which satisfies this condition. The resulting temperature distribution is given below and shown in Fig. 3

$$\theta = \Phi \frac{1 + R_B + 2S(1 - R_B)}{1 - (R_B - 1)S} \left\{ \left[\frac{1 + R_B + S(1 - R_B)}{1 + R_B + 2S(1 - R_B)} \right] \eta - \frac{1}{2}\eta^2 + \frac{1}{3} \left[\frac{S(1 - R_B)}{1 + R_B + 2S(1 - R_B)} \right] \eta^3 \right\} \quad (13)$$

It is evident that in this case the energy absorbed by the diffuser from the radiation field must be conducted out the front surface for energy conservation. Hence the temperature distributions for this case are all monotonic.

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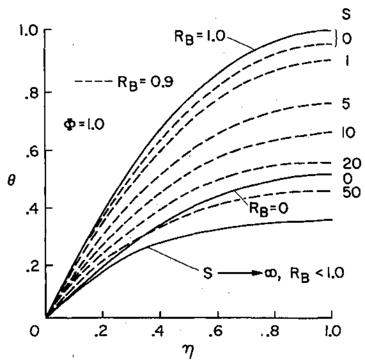

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Influence of a Temperature Dependent Spectral Absorption Coefficient on Radiative Flux

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Introduction

THIS Note reports the results of an investigation of the influence of a temperature dependent absorption coefficient on the continuum radiative flux from a nonisothermal atomic hydrogen shock layer. In a previous paper,[1] the nongray absorption coefficient was assumed to be independent of location in the shock layer and was evaluated at the temperature just behind the shock. Actually, experimental and theoretical work[2] reveals that the absorption coefficient of an atomic gas is a strong function of temperature and, hence, location in the shock layer. In some radiative transfer studies, this presents no difficulty because the shock layer plasma is assumed to be isothermal. Many nonisothermal investigations have included a temperature dependent absorption coefficient, but the necessity of including the temperature dependence in radiative flux calculations has not been investigated.

The analysis is developed in Ref. 1 and is based on the following assumptions: 1) local thermodynamic and chemical equilibrium, 2) one-dimensional, radiative energy transport, 3) negligible radiation emitted from the body, and 4) negligible precursor effects. Line radiation (bound-bound) and the influence of stimulated emission are not considered.

The actual temperature profile in the shock layer can be determined from the coupled conservation equations. Because the objective of the present study is to investigate nongray-nonisothermal radiation, the fluid dynamics and radiation are uncoupled by assuming a linear temperature profile in the shock layer, $T/T_r = \theta(\bar{x}) = \theta_o + (1 - \theta_o)\bar{x}$. This temperature distribution represents the actual temperature profile more accurately than the isothermal approximation, which has been used in many other investigations. The shock wave is located at $\bar{x} = 1$ and the body at $\bar{x} = 0$. The reference temperature T_r is a function of the ambient density and Mach number and is given by the Rankine-Hugoniot equations, and θ_o is the nondimensional temperature at $\bar{x} = 0$, which is determined by the body's ablating properties.

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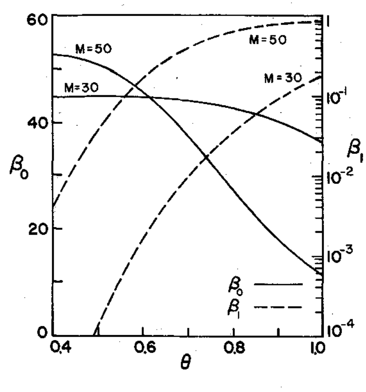

Fig. 1 Variation of β_0 and β_1 with temperature.

Using the notation of Ref. 1 the continuum absorption coefficient for an $m + 1$ level hydrogen atom is given by[2]

$$\varkappa_\nu(T) = \sum_{k=0}^{m} \beta_k(T)\alpha_k(\nu) \quad 0 < \nu < \infty \tag{1}$$

The frequency variation of the ith electronic state is

$$\alpha_i(\nu) = 0 \quad \nu < \nu_i$$

$$\alpha_i(\nu) = (\nu_i/\nu)^3 \quad \nu > \nu_i \tag{2}$$

where ν_i is the ionization edge of the ith electronic state. The ground state corresponds to $i = 0$, while the first excited state corresponds to $i = 1$. Assuming the ith electronic state influences the absorption coefficient only in the spectral interval, $\nu_i \leqq \nu < \nu_{i-1}$, the radiative flux becomes

$$F = \sum_{i=0}^{m} F_i \tag{3}$$

where F_i is the flux from the ith electronic state. Because the absorption coefficient is a separable function of position and frequency, the dimensionless radiative flux incident on the body becomes

$$\bar{F}(0) = \frac{F_i(0)}{\sigma T_r^4} = \frac{-30}{\pi^4}\int_{\bar{\nu}_i}^{\infty}\int_o^{\tau_i} \frac{\bar{\nu}^3\alpha_i(\bar{\nu})E_2[\alpha(\bar{\nu})t]\,dt\,d\bar{\nu}}{\exp[\bar{\nu}/\theta(t)]-1}, \tag{4}$$

where the dimensionless frequency is $\bar{\nu} = h\nu/kT_r$ and the optical thickness is

$$\tau_i = \int_o^L \beta_i(T)\,dx$$

Table 1 Reference conditions

Ambient temperature = 300°K
Ambient pressure = 1 cm Hg
Ambient density = 1.087×10^{-6} g/cm^3

Rankine Hugoniot solutions

M	T_r
26	14,251
30	16,053
34	17,705
38	19,340
42	21,067
46	23,036
50	25,530

Values of τ_o (L in cm units)

	Constant β ($\theta_o = 1.0$)	Variable β $\theta_o = 0.5$	Variable β $\theta_o = 0.0$
$M = 30$	$36.1L$	$42.3L$	$43.6L$
$M = 50$	$11.2L$	$32.1L$	$42.6L$

Values of τ_1 (L in cm units)

	Constant β ($\theta_o = 1.0$)	Variable β $\theta_o = 0.5$	Variable β $\theta_o = 0.0$
$M = 30$	$0.181L$	$0.0428L$	$0.0214L$
$M = 50$	$0.852L$	$0.501L$	$0.252L$

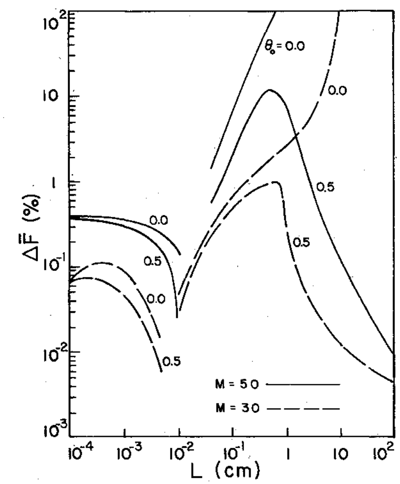

Fig. 2 Flux difference between variable β_o and constant β_o assumption.

The shock layer thickness is denoted by L. Table 1 gives T_r, τ_o, and τ_1 for several Mach numbers.

When only the ground state exists, $\beta_o(T)$ is equal to the ground state photo-ionization cross section times the hydrogen atom population. When both ground state and the first excited state exist, $\beta_o(T)$ is equal to the ground state photoionization cross section times the ground state population, and $\beta_1(T)$ is equal to the first excited state photoionization cross section times the first excited state population. The cross sections are taken from Zeldovich and Raizer.[2] Figure 1 shows $\beta_o(T)$ and $\beta_1(T)$ for Mach 30 and Mach 50. When the ground and the first excited state are allowed to coexist, $\beta_o(T)$ remains essentially the same as when only the ground state is considered for the thermodynamic conditions considered herein.

Results and Discussion

The temperature independent absorption coefficient approximation is equivalent to assuming $\beta_i(T)$ to be constant at its value just behind the shock wave, $\beta_i(T_r)$. The constant β assumption influences the radiative flux through τ_i. The ground state optical thickness, τ_o is decreased, because the ground state population is much less in the hot plasma near the shock wave than it is in the relatively cool gas near the body. The opposite variation occurs for τ_1, since the excited state population is a maximum near the shock and a minimum near the body. The current study considers only the Lyman and Balmer continuum ($m = 1$) for the ambient conditions listed in Table 1.

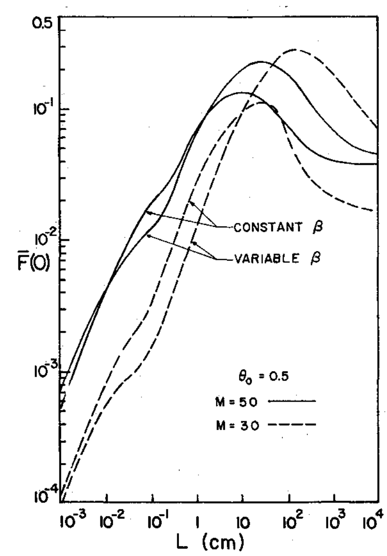

Fig. 3 Flux from hydrogen plasmas with variable β and constant β assumption.

Influence of variable β

The accuracy of the constant β assumption is investigated[3] by calculating the flux to the body with constant β and comparing it with the current results (variable β). The results are presented for ground state radiation only in Fig. 2 which shows the absolute value of the difference between the constant β_o flux and variable β_o flux divided by the variable β_o flux as a function of the shock-layer thickness for Mach 30 and Mach 50. When the shock-layer is optically thin, the variable β_o flux is greater than the constant β_o flux. As the shock layer becomes optically thick, the constant β_o flux becomes greater than the variable β_o flux. The constant β_o flux and variable β_o flux become equal near an optical thickness of unity.

When τ_o is small, the flux is proportional to τ_o; consequently, the variable β_o flux is greater than the constant β_o flux because the variable β_o shock layers have larger optical thicknesses. When τ_o becomes large, attenuation becomes important. Thus, the variable β_o flux is less than the constant β_o flux because the optical thickness of the variable β_o shock layers is greater. One can conclude from Fig. 2 that for shock layer thicknesses of less than 0.1 cm, the error in the constant β_o approximation is less than one percent. As the thickness increases and θ_o decreases, the error in the constant β_o approximation increases.

Influence of shock layer thickness

Figure 3 gives the total flux to the body from the ground state and the excited state as a function of the shock layer thickness for Mach 30 and Mach 50, respectively. The ground state flux is larger than the excited state flux for small values of the shock layer thickness L. As L increases, the ground state flux increases and reaches its maximum and then falls off rapidly because of self-absorption and the temperature profile. It quickly becomes negligible with respect to the excited state flux. The exchange between the ground state and the excited state as the dominant source of radiation appears as a rapid change of slope near $L = 0.1$ cm. As L continues to increase, the excited state flux reaches its maximum and begins to fall off due to the same processes.

Figure 3 shows that the maximum ground state flux for the constant β assumption occurs at slightly larger plasma thicknesses than it does for variable β. For the excited state flux the maximum for the constant β assumption occurs at slightly smaller plasma thicknesses than it does for the variable β case.

For large L the constant β_1 assumption overestimates self-absorption. The local temperature determines the excited state population; therefore, the variable β_1 plasma has a cool region where its excited state is optically thin. Hence, more of the radiation emitted near the shock wave reaches the body for the variable β_1 case, than for the constant β_1 case.

At small shock layer thicknesses, the assumption of constant β gives a better approximation to the total flux than it does for the ground and excited state fluxes individually because they vary in opposite directions cancelling out the error. As L becomes large, the flux becomes almost entirely due to the excited state so that the assumption of constant β introduces errors of the same magnitude as it does for the excited state alone. One can conclude from this analysis that the constant β approximation gives results that show the same trends and roughly the same magnitudes as the variable β calculations.

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Measurements of Skin Friction on the Wall of a Hypersonic Nozzle

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Nomenclature

C_f = local skin-friction coefficient, $2\tau_w/\rho_e u_e^2$
M = Mach number
p = pressure
R_e = momentum thickness Reynolds number, $\theta\rho_e u_e/\mu_e$
T = temperature
u = velocity
x = axial distance from nozzle throat
θ = momentum thickness
τ = shear stress
ρ = density
μ = viscosity

Subscripts

aw = adiabatic temperature
c = calculated
e = boundary-layer edge
w = wall
0 = settling chamber

Introduction

COMPARISONS of mean profile data in turbulent boundary-layers on the walls of hypersonic nozzles have been made for a wide range of Mach numbers.[1] In general, boundary-layers on nozzle walls are subjected to streamwise pressure and temperature gradients during their initial development. As a result of these gradients, a quadratic variation of total temperature with velocity occurs in the outer part of the boundary-layer.[2] The quadratic variation is typical of cold-wall nozzle boundary layers while a nearly linear variation occurs in most flat plate boundary-layers.[3] Comparisons[4–6] of nozzle-wall and flat-plate skin-friction data with commonly used prediction methods have indicated that surface shear-stress and heating may not be strongly influenced by upstream history. Recent reviews by Beckwith[7] and Cary[8] suggest a need for accurate skin-friction and heat-transfer data at hypersonic Mach numbers and cold-wall conditions in order to test the validity of various prediction methods for such conditions.

To help satisfy this need, direct measurements of skin friction have been obtained on the wall of a Mach 19.8 nozzle. Mean profile data in the turbulent boundary layer on the wall of this nozzle have been published.[1] The new direct measurements of

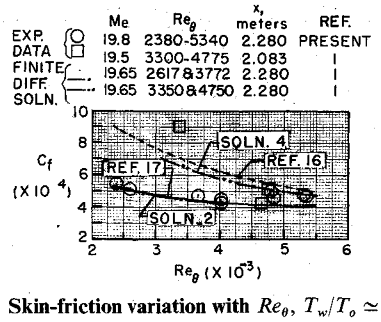

Fig. 1 Skin-friction variation with Re_θ, $T_w/T_o \simeq 0.18$.

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Index category: Fluid Dynamics.

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